Lecture 7: Matrix Transpose

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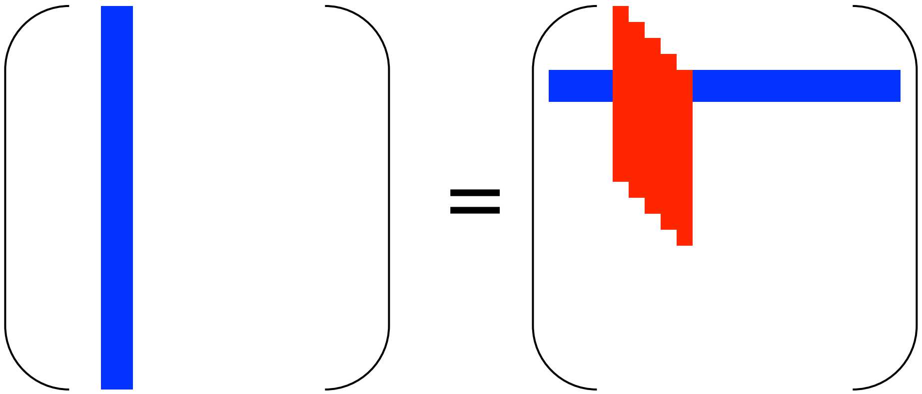
Simple Example: Matrix Transpose

- ```
  do j=1,n
     do i=1,n
        b(i,j) = a(j,i)
     enddo
  enddo
  ```



- No temporal locality (data used once)
- Spatial locality only if (words/cacheline) * n fits in cache
  - Otherwise, each column of a may be read (words/ cacheline) times
  - Transpose is *semilocal* at best

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  ```

Performance Models

- What is the performance model for transpose?
  - N^2 loads and N^2 stores
  - Simple model predicts STREAM performance
    - Its just a copy, after all

Example Results

Matrix Size	Time
100x100	4700 MB/s
400x400	1200 MB/s
2000x2000	705 MB/s
8000x8000	*did not complete

- Why is the performance so low?
  - Compiler fails to manage spatial locality in the large matrix cases
  - Why does performance collapse at 8000x8000 matrix
    - May seem large, but requires 1GB of memory
    - Should fit into main memory
- What might be done to improve performance?

Question

- Model the performance of a transpose with this simple model:
  - ♦ Assume that the size of the cache is just a few cachelines. Then
    - Access to consecutive elements in memory will read from the cacheline (spatial locality)
    - Access to nonconsecutive elements in memory (the b array in our example) will not be in the available cachelines, forcing a full cacheline to be accessed for every store. Assume a cacheline stores 64 bytes.
  - ♦ What is the time cost of a transpose with this model? Use the STREAM performance data as the sustained memory performance in moving data to or from memory to cache

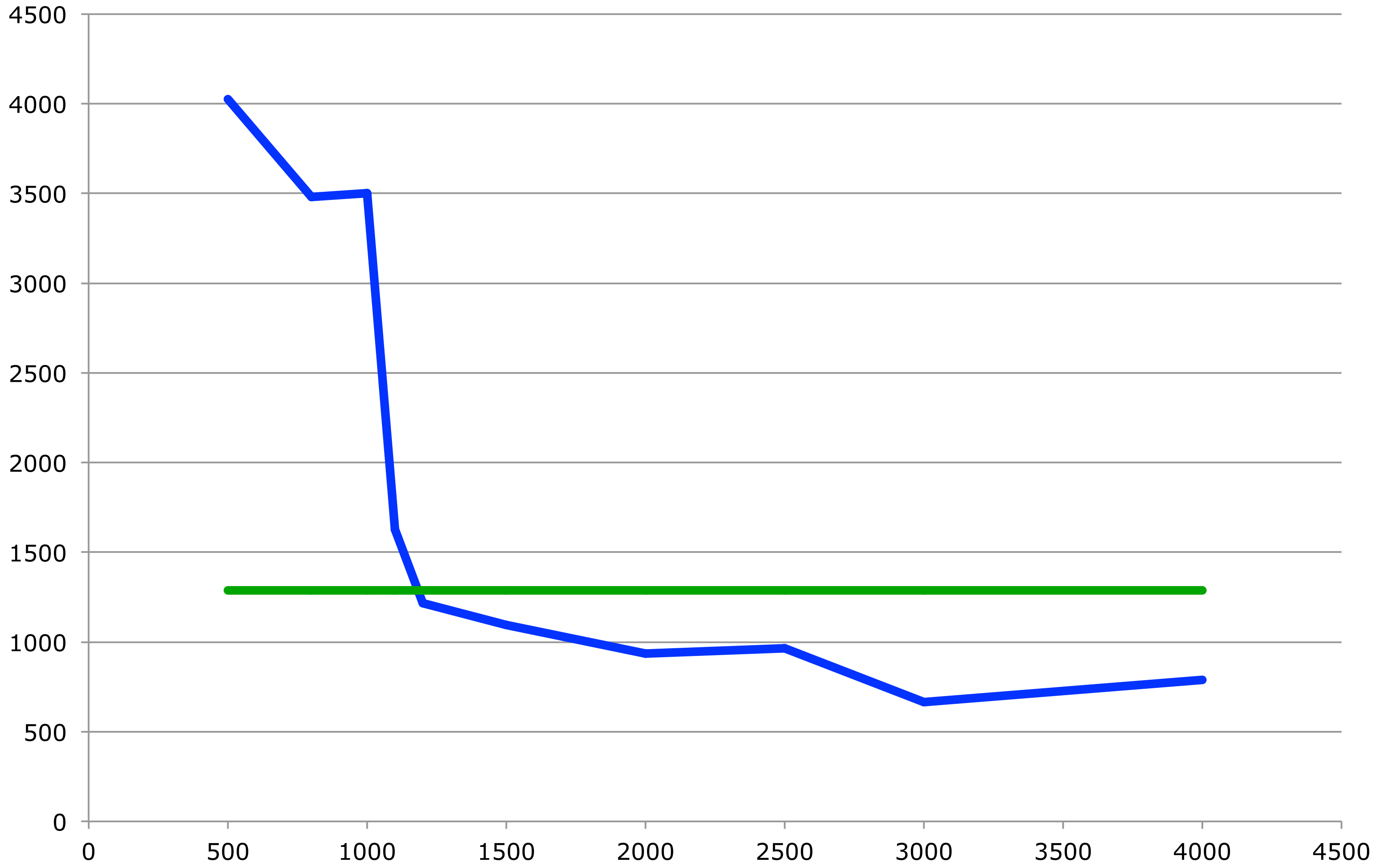
A Simple Performance Model for Transpose

- If source and destination matrices fit in cache, then
  - $T = n^2(r_c + w_c)$
- If the source and destination matrices do not fit in cache
  - $T = n^2(r + Lw)$
  - Where L is the number of elements per cacheline.
- Note that these are not sharp predictions but (approximate) bounds

Lets Look at One Case

- My Laptop
- STREAM performance in Fortran, for 20,000,000 element array
  - 11,580 MB/sec
- Simple Fortran transpose test
  - gfortran –o trans –O1 trans.f
  - Low optimization to avoid "smart compiler" issues with this demonstration
- Performance bound (model):
  - Assume r = w = 1/11,580e6
  - $T=n^2(r+8w) = n^2(9r)$
  - Rate = $n^2/T = 1/9r$

Transpose Performance



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Observations

- Cache effect is obvious
  - ♦ Performance plummets after n=1000
  - ♦ Need to hold at least one row of target matrix to get spatial locality
    - N * L bytes (64k for N=1000, L=64 bytes)
- STREAM estimate gives reasonable but not tight bound
- Achievable performance for the operation (transpose) is much higher (effectively COPY)

Yes Another Complication

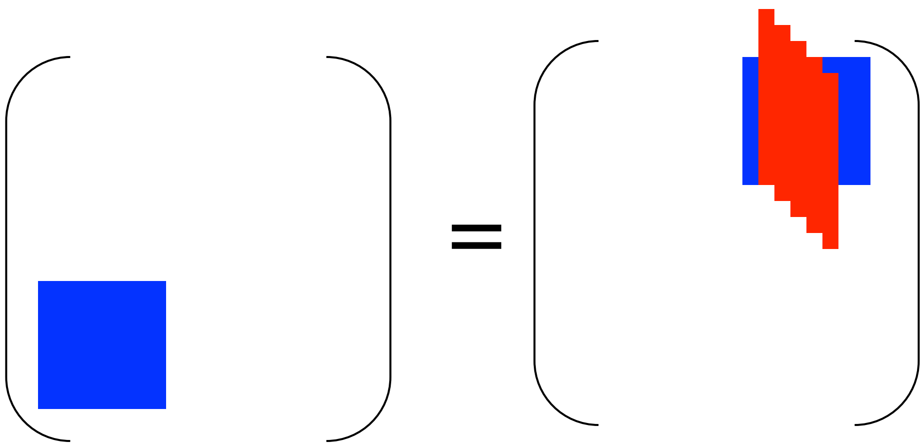
- How many loads and stores from memory are required by a=b?
  - ♦ Natural answer is
    - One load (b), one store (a)
- For cache-based systems, the answer may be
  - ♦ Two loads: Cacheline containing b *and* cacheline containing a
  - ♦ One store: Cacheline containing a
  - ♦ Sometimes called *write allocate*

And Another Complication

- When do writes from cache back to memory occur
  - When the store happens (i.e., immediately)
    - This is called “write through”
    - Simplifies issues with multicore designs
    - Increases amount of data written to memory
  - When the cache line is needed
    - This is called “write back”
    - Reduces amount of data written to memory
    - Complicates hardware in multicore designs
- “Server” systems tend to have write-back; lower performance systems have write-through

Loop Transformations

- Reorder the operations so that spatial locality is preserved



- Break loops into blocks
  - Strip mining
  - Loop reordering

Strip Mining

- Break a loop up into blocks of consecutive elements
- Do k=1,n
  ```
      a(k) = f(k)
  enddo
  ```
- Becomes
  ```
  do kk=1, n, stride
      do k=kk,min(n,kk+stride-1)
          a(k) = f(k)
      enddo
  enddo
  ```
- For C programmers, do k=1,n,stride is like for(k=1; k<n; k+=stride)

Strip Mining

- Applied to both loops in the transpose code,
- ```
  do j=1,n
      do i=1,n
  ```
  Becomes
  ```
  do jj=1,n,stride
      do j=jj,min(n,jj+stride-1)

          do ii=1,n,stride
              do i=ii,min(n,ii+stride-1)
  ```
- Still the same access pattern, so we need another step …
  ```

Loop Reordering

- Move the loop over j inside the ii loop:

```
do jj=1,n,stride
    do ii=1,n,stride
        do j=jj,min(n,jj+stride-1)
            do i=ii,min(n,ii+stride-1)
                b(i,j) = a(j,i)
```

- Value of stride chosen to fit in cache
  - Repeat the process for each level of cache that is smaller than the matrices
    - Even a 1000 x 1000 matrix is 8 MB, = 16MB for both A and B. Typical commodity processor L2 is 2MB or smaller, so even modest matrices need to be blocked for both L1 and L2

Multiple levels of Cache

- Blocking is not free
  - ♦ There is overhead with each extra loop, and with each block
    - Implies that blocks should be as large as possible and still ensure spatial locality
- Moving data between each level of cache is not free
  - ♦ Blocking for each level of cache may be valuable
  - ♦ Block sizes must be selected for each level

Example Times for Matrix Transpose

5000x5000 transpose (a *very* large matrix)	Unblocked	L1 Blocked	L1/L2 Blocked
(20,100,g77)	2.6	0.55	0.46
(32,256,g77)	2.6	0.46	0.42
(32,256,pgf77,main)	0.58	0.48	0.55
Same, within a subroutine	2.8	0.55	0.48

Observations

- Blocking for fast (L1) cache provides significant benefit
- Smart compilers can make this transformations
  - See pgf77 results
- But only if they have enough information about the data
  - When the array passed into a routine instead of everything in the main program, results no better than g77
- Parameters are many and models are (often) not accurate enough to select parameters

Why Won't The Compiler Do This?

- Blocking adds overhead
  - More operations required
- Best parameter values (stride) not always easy to select
  - May need a different stride for the I and the J loop
- Thus
  - Best code depends on problem size, for small problems, simplest code is best
- Notes some compilers support annotations to perform particular transformations, such as loop unrolling, or to provide input on loop sizes (the "n")

Why Don't Programmers Do This?

- Same reason compilers often don't – not easy, not always beneficial
- But you have an advantage
  - You can form a performance expectation and compare it to what you find in the code
    - Measure!
  - You often know more about the loop ranges (n in the transpose)
- This is still hard. Is there a better way?
  - Sort of. We'll cover that in the next lecture.

Questions

- Develop a performance bound for this operation
  - ```
    do i=1,n
        a(i*stride) = b(i)
    enddo
    ```
  - How does your model depend on stride?
  - What changes in your model if the cache uses a write-allocate strategy?
  - What changes if the copy is
    ```
    do i=1,n
        a(i) = b(i+stride)
    enddo
    ```
- Note: such a “strided copy” is not uncommon and may be optimized by the hardware
  - This model does not take that into account
    ```

Question

- In blocking the transpose, we used the same block size for the rows and column. Is this necessary? Why might a different value make sense?